Section 9.2 Exponential Functions

In this math course we have been working with polynomial expressions and functions that have been defined to be some arithmetic combination of numbers and variables. There are many other kinds of functions.

An exponential function has the variable in the exponent position rather than the base position. For example $f(x) = 2^x$.

$f(1) = 2$ $f(2) = 2^2 = 4$ $f(3) = 2^3 = 8$ $f(0) = ?$ $f(-1) = \frac{1}{2}$ $f(-2) = \frac{1}{4}$	

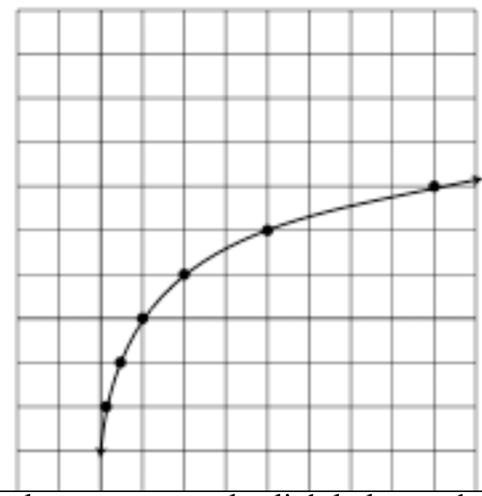
This curve is said to be "exponential" meaning that it grows ***very*** fast. As x gets larger, y gets much larger than it was before.

If we had the equation $g(x) = 3^x$, the coordinate (0,1) would be common to both. g will get much larger than f for the same x values.

A common application of exponential functions are in computing compound interest. Compound interest means that each time the interest is computed, it is added to the principal. When the interest is again computed, it is calculated not only on the original principal but on the accumulated interest as well.

Consider $x = 2^y$. Here the x and y are reversed. We will make a plot to see what this looks like:

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3



This curve is said to be "logarithmic". As *x* gets larger, *y* gets only slightly larger than it was before.

Section 9.3 Logarithmic Functions

Given $y = 2^x$. We have seen its graph in the previous section. We also know how to find the ***inverse function***. We exchange the x and y and then solve for y.

We find the inverse to be: $\begin{matrix} y = 2^x \\ x = 2^y \end{matrix}$ but we have a problem. How do we get the y out of the exponent position?

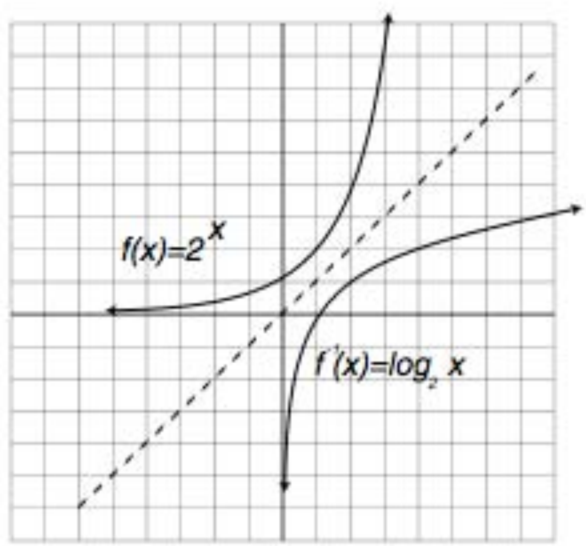
Mathematicians defined a new function that looks like this: $\log_2 x$ that means "the exponent to which we raise 2 to get x."

$\log_2 x$ is read: "the log, base 2, of x". Some people read it: "the logarithm, base 2, of x." No matter how it is read, it defines an **exponent**.

A more useful way to remember the relationship *without* becoming confused is:

☆ $\log_b a = x$ implies $b^x = a$

Finishing our original example….The inverse function is $y = \log_2 x$. The graph of this (or any) inverse function, is a reflection of the graph across the line $y = x$.:



Notice that the original function has coordinates (0,1), (1,2) and (2,4) and the inverse has (1,0), (2,1) and (4,2). The x and y values are exchanged.

Also notice that both are, in fact, functions. Each does pass both the vertical and horizontal line tests.

Examples This problem is easier to do as an exponent than as a log so use ☆

$\log_2 32 = x$

$32 = 2^x$

$2^5 = 2^x$ *so* $5 = x$

$\log_2 1 = x$

$2^x = 1$ *so* $x = 0$

$\log_2 \frac{1}{8} = x$

$2^x = \frac{1}{8}$

$2^x = 2^{-3}$ *so* $x = -3$

Logs are the inverse functions for exponentials.

If $f(x) = 5^x$ then $f^{-1}(x) = \log_5 x$

$f(3) = 5^3$ then $f^{-1}(125) = \log_5 125$

$= 125$ $\log_5 125 \overset{set}{=} a$

$5^a = 125$

$5^a = 5^3$ *so* $a = 3$

simplify: $7^{\log_7 85}$

$\log_7 85 \overset{set}{=} a$

$7^a = 85$ by definition of logs

So, if 7^a is 85 and $a = \log_7 85$, then $7^{\log_7 85}$ must also equal 85.

Notice the bases are the same! The exponential "Seven to the a" undoes Log base 7 of 85 giving us 85.

Section 9.4 Properties of Logarithmic Functions

$$\log_b a = x \Leftrightarrow b^x = a$$

Based on the above relationship, a logarithm is an exponent so Log properties are based on the properties of exponents.

$$x^p \cdot x^q = x^{p+q}$$

When two numbers with the same base (in the above example, x) are multiplied, you add their exponents. Translated to logarithms, $\log_b(xy) = \log_b x + \log_b y$.

$$\log_b(x \cdot x) = \log_b x + \log_b x$$
$$\log_b x^2 \quad = 2\log_b x$$

Consider….

$$\log_b(x \cdot x \cdot x) = \log_b x + \log_b x + \log_b x$$
$$\log_b x^3 \quad = 3\log_b x$$

this leads us to $\log_b x^n = n\log_b x$

This is an important issue. In the process of finding the inverse function, we change the letter x to z and change function name or y to x. Finally, we solve for z to get the inverse function. It is this "solving for z" that causes us trouble. We need logarithms because they define how to "move an exponent down to become a factor:

$y = 2^x$	original function
$x = 2^z$	exchange letters
$\log(x) = \log(2^z)$	take log of both sides
$\log(x) = z\log 2$	Use log property
$\dfrac{\log(x)}{\log 2} = z$	divide by log 2
$\dfrac{\log(x)}{\log 2} = y$	This is the inverse function of the original

Consider ….
$$\frac{x^p}{x^q} = x^{p-q}$$
This suggests subtraction.... $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

What would happen if we let $y = x$?
$$\log_b\left(\frac{x}{x}\right) = \log_b x - \log_b x$$
$$\log_b(1) = 0$$

$1 = x^0$ Of course, x to the zero power is one!

What about $\log_b(b)$? We let x be our unknown. Using the definition of logs, ($\log_b a = x \Leftrightarrow b^x = a$) the equation we get gives us
$$\log_b(b) = x$$
$$b^x = b$$
$$b^x = b^1$$
. So $x = 1$!

This is an important fact. **If the base of the log is the same as the argument of the log (as in this case), the value of the log is ONE!**

$\log_{17} 17 = ?$

$? = 1$

$\log_2(4 \cdot 16)$

Method 1	Method 2	Method 3
$\log_2 64$	$\log_2 4 + \log_2 16$	$\log_2 2^2 + \log_2 2^4$
$\log_2 2^6$	$2 + 4$	$2\log_2 2 + 4\log_2 2$
$6\log_2 2$	6	$2 + 4$
6	6	6